<u>The kinematics equations</u> (the equations that describe an object's motion)

For those of you in high school, there may seem to be a huge gap between the motion equations that you've studied and those used in the simulations. Unfortunately, most high school texts work only in 1-dimension and at best use the equation: $\mathbf{d = v_i\, t + (1/2)\, a\, t^2}$.

[In case you've forgotten, "**d**" is the distance travelled (assuming the object started at the origin), "$\mathbf{v_i}$" is the initial speed, "**a**" is the acceleration, and "**t**" is the time from the start of the problem.]

At worst, high school texts sometimes present two equations: $\mathbf{d = v\, t}$, and $\mathbf{s = (1/2)\, a\, t^2}$, where "**s**" is also distance travelled. The use of "**d**" and "**s**" avoids acknowledging the directional aspect of most of these quantities (other than time "**t**" and mass "**m**").

The general kinematics equation, or **position function**, used in the simulations is:

$$\mathbf{x_f = x_o + v_{ox}\, t + (1/2)\, a_x\, t^2}$$

where "$\mathbf{x_f}$" is the final location **--** it's more formally given as **x(t)** to emphasize the functional nature of the equation. "$\mathbf{x_o}$" (read as x zero) is the position of the object at t = 0 and is necessary because the object may not initially be at the origin; it's sometimes written as "$\mathbf{x_i}$" where the "**i**" stands for "initial". "$\mathbf{v_{ox}}$" is the component of velocity in the x direction at t = 0 and is often referred to as the initial velocity, and sometimes written as $\mathbf{v_{ix}}$. Together $\mathbf{x_o}$ and $\mathbf{v_{ox}}$ are known as the **initial conditions**.

We understand that net force will cause changes to an object's motion, so why isn't it present in the position function? Well, it is there "in disguise" represented by its effect -- the acceleration. Instead of writing the 2nd order term as: $\mathbf{(1/2)\, (F_{net}/m)\, t^2}$, we prefer to use $\mathbf{(1/2)\, a_x\, t^2}$ which is more compact and also allows us to speak of the effect – the acceleration, as its own entity. [Refer to the Newton's Laws link for information about $\mathbf{a_x = \Sigma F_x/m}$.]

The **position function** shows that the final location of an object is composed of 3 contributions: $\mathbf{x_o}$, which is where it was to begin with, $\mathbf{v_{ox}\, t}$, which is the distance travelled due to its initial velocity, and $\mathbf{(1/2)\, a_x\, t^2}$ which is the distance travelled due to the net force changing its speed.

If the object is moving in two dimensions there would also be a corresponding equation for the y direction with each x replaced by a "y" :

$$\mathbf{y_f = y_o + v_{oy}\, t + (1/2)\, a_y\, t^2}$$

The two high school equations mentioned earlier are just special cases of the position function:

if $\mathbf{ax} = \mathbf{0}$, and $\mathbf{x_0 = 0}$, then $\mathbf{x_f = v_{ox}\, t}$, which is simply the formula $\mathbf{d = v\ t}$. Such situations are referred as <u>uniform motion</u> where uniform means constant velocity and direction.

if $\mathbf{v_{ox} = 0}$, and $\mathbf{x_0 = 0}$, **then** $\mathbf{x_f = (1/2)\, a_x\, t^2}$, which is $\mathbf{s = (1/2)\, a\, t^2}$. Such situations are referred to as <u>uniform acceleration</u> with uniform again denoting constant acceleration.

Neither of these two simplified equations allows for an object to be anywhere other than the origin at t = 0.

The **velocity function** of an object can be obtained from the position function by taking the time derivative:

$$v_{fx} = v_{ox} + a_x\, t \qquad v_{fy} = v_{oy} + a_y\, t$$

Although two other kinematics equations exist, neither is necessary for the simulations.

The **velocity function** indicates that the final velocity of an object is composed of 2 contributions: $\mathbf{v_{ox}}$, which is how fast it was moving to begin with, and $\mathbf{a_x\, t}$ which is the change in speed due to the net force acting on the object's inertia.

Caution: The kinematics equations are <u>only valid if the acceleration is constant</u>, <u>meaning the net force and mass must also be constant</u>.